2024 AMS SECTIONAL MEETING

Modeling multi-state health transitions with a self-exciting process

Jiwon Jung
Ph.D. Student
Department of Statistics
Purdue University

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Happy Birthday!

Collaborators

Prof. Kiseop Lee
Department of Statistics
Purdue University

Prof. Mengyi Xu
Department of Statistics
Department of Mathematics
Purdue University

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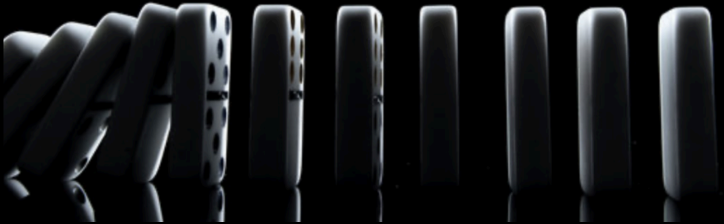
Introduction

Movements with Momentum

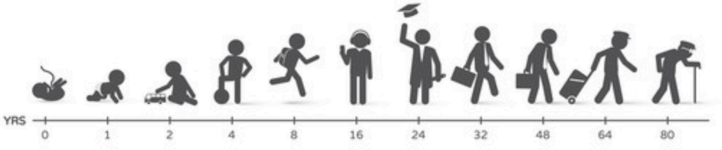


- Most stochastic models used in quantitative finance and insurance assume the Markov property because of its mathematical tractability.
- One commonly observed phenomenon violating the Poisson arrival as well as the Markov assumption is the momentum effect.

Beyond the Markov Models

- Does the concept of the “momentum effect” apply to health transition dynamics?
- To capture this momentum effect, what alternative methods can we use?

Introduction



- Understanding the dynamics of health transition is crucial for pricing aged care products effectively in the evolving health market.

Previous Studies

- Much research on multi-state health transition models has relied on the **Markov property**, where future states depend only on the current state, irrespective of past history.
  - Fong et al. (2015) proposed using a generalized linear model to estimate age- and sex-specific transition rates.
  - Hanewald et al. (2019) adapted this approach to include deterministic time trends.
  - Li et al. (2017) and Sherris and Wei (2021) expanded it into a stochastic model using a multi-state latent factor intensity model to account for systematic trends and uncertainties in health transitions.
- Research demonstrates that probabilities of functional status transitions are duration-dependent. This line of study employs **semi-Markov process** models, which consider age, current status, and duration in the current state.
  - Hardy and Gill (2005), Hardy et al. (2006), Cai et al. (2006), and Biessy (2017) have investigated this duration dependency in future transitions.
  - However, the state and duration effect with respect to the past functional disability experience has been less studied.

Motivation

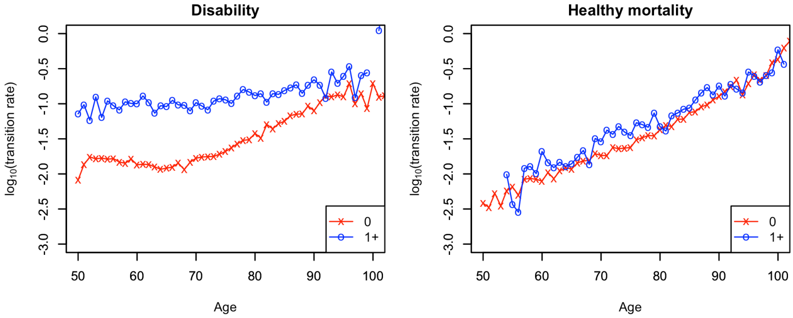
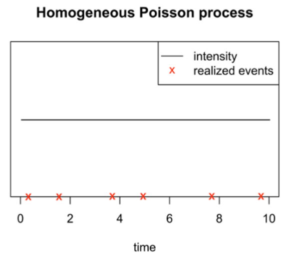


Figure 1. Crude health transition rates with respect to the number of past functional disabilities.

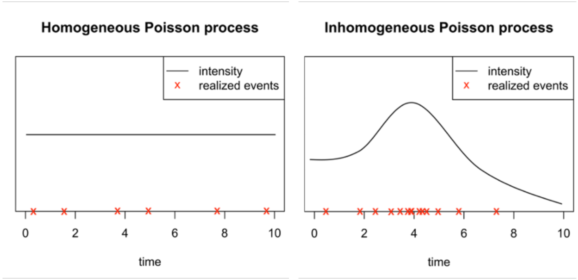
- Our explanatory data analysis suggests that the elderly with prior functional disabilities are at higher risk of experiencing it again and have higher mortality rates than those without a history of disability.

Backgrounds

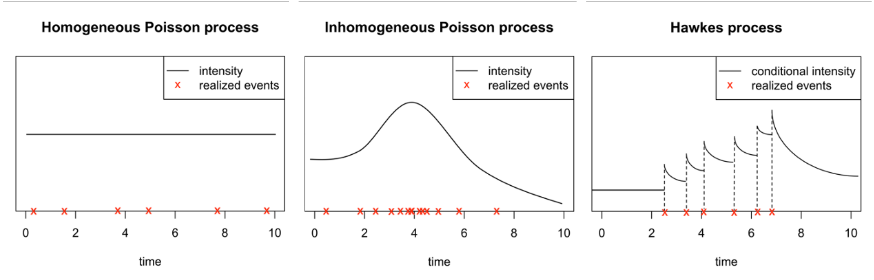
Momentum and a Hawkes Process



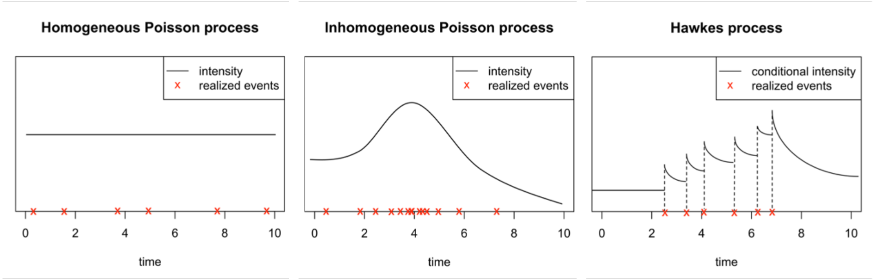
Momentum and a Hawkes Process



Momentum and a Hawkes Process



Momentum and a Hawkes Process



- A counting process with a stochastic intensity is called a doubly stochastic Poisson process.
- A Hawkes process (Hawkes, 1971) is a popular doubly stochastic process with *self-exciting* properties; *an event occurrence increases the probability of the occurrence of another event*.

Momentum and a Hawkes Process

Definition

A Hawkes process is a point process $N(t)$ which is characterized by its conditional intensity $\lambda(t)$ with respect to its natural filtration:

$$\lambda(t|\mathcal{F}_{t-}) = \phi(t) + \int_0^t \mu(t-s)\mathrm{d}N(s), \tag{1}$$

where $\phi(t)$ is the background intensity function, and the $\mu(t)$ is the excitation function satisfying $\int_0^\infty \mu(s)\mathrm{d}s < 1$.

- Hawkes processes model self-exciting properties in diverse fields:
  - Finance: **hawkes2018hawkes**; **da2017correlation**
  - Insurance: **JungLeeXu**; Swishchuk et al. (2021)
  - Epidemiology: **browning2021simple**

Goal

- Our goal is to estimate the intensity of age and gender-specific transitions by incorporating the impact of the past functional disability as well as time spent in the current state using a self-exciting process.

Four-State Health Transition Model

Four-State Health Transition Model I

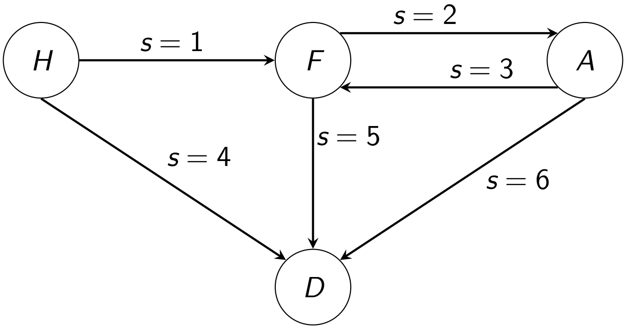


Figure 2. The four-state health state transition model. H means healthy; F means functionally disabled or simply disabled; A means reactivated; D means dead. The notation s represents the type of transitions.

Four-State Health Transition Model II

- The transition intensity for individual k of transition type $s \in \{1,2,3,4,5,6\}$ at time t is given by

$$\lambda_s(t) = \underbrace{\phi_s(t)}_{\text{background intensity}} + \underbrace{\mu_s(t - T_t)}_{\text{exciting function}} \cdot \underbrace{\mathbb{1}_F(t)}_{\text{disability indicator}}$$

  - $\phi_s(t)$ captures the impact of observable variates such as the (scaled) age $x_k(t)$ and the gender indicator F_k at time t.
    - $\phi_s(t) = \exp(\beta_s^{intercept} + \beta_s^{age} x_k(t) + \beta_s^{female} F_k)$
    - $\phi_1(t) = \phi_3(t)$ and $\phi_4(t) = \phi_6(t)$
  - $\mu_s(\cdot)$ captures the impact of the past functional disability and the duration in the current state ($t - T_t$, where T_t is the latest transition time).
    - $\mathbb{1}_F(t) = 0$ if in the healthy state at time t.
    - $\lambda_1(t) = \phi_1(t)$ and $\lambda_4(t) = \phi_4(t)$.

Four-State Health Transition Model III

- Choice of Hawkes kernels $\mu_s(\cdot)$:
  - Exponential kernel (monotonic decay):

$$\mu_s(x) = \alpha_s \, e^{-\delta_s x}, \quad \alpha_s \geq 0, \delta_s > 0, \alpha_s < \delta_s.$$

  - Rayleigh kernel (non-monotonic decay):

$$\mu_s(x) = \theta_s(x + \kappa_s) \, e^{-\eta_s(x+\kappa_s)^2/2}, \quad \theta_s \geq 0, \eta_s > 0, \kappa_s > 0, \theta_s < \eta_s.$$

Data Preparation I

- We use the `RAND HRS Data` 1992-2018 from the U.S. Health and Retirement Study (HRS), a nationally representative longitudinal panel survey.[1]
- The HRS is a biennial survey which began in 1992 and follows up with interviews of initially non-institutionalised Americans aged 50 and above.
- The health state is determined by a person's ability to perform activities of daily living (ADLs), such as bathing, toileting, and dressing.

[1] https://hrs.isr.umich.edu/data-products

Data Preparation II

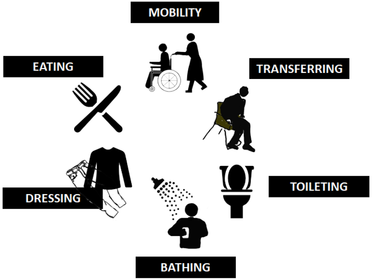


Figure 3. Six activities of daily livings (ADLs) (credit: **adl**)

- Two or more ADL dependencies indicate functional disability, in line with long-term care insurers' practice.

Estimation

Maximum Likelihood Estimation

Suppose there are a total of K individuals, S transition types, and J interview waves. The complete log likelihood function is given by

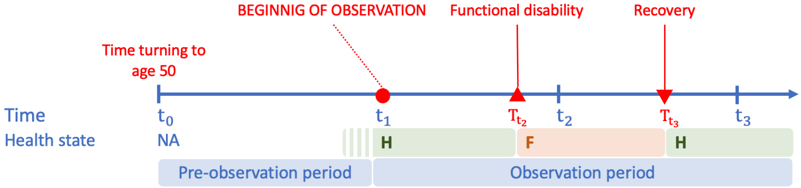
$$l(\boldsymbol{\theta}) = \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{j=1}^{J-1} l_{k,s,j}(\boldsymbol{\theta}), \tag{2}$$

where $\boldsymbol{\theta}$ denotes the set of parameters to be estimated, and

$$\begin{aligned} l_{k,s,j}(\boldsymbol{\theta}) = Y_{k,s,j} \ln \lambda_{k,s}(\hat{t}_{k,j}) &- R_{k,s}(t_{k,j}) \int_{t_{k,j}}^{\min\{\hat{t}_{k,j}, t_{k,j+1}\}} \lambda_{k,s}(u)\mathrm{d}u \\ &- R_{k,s}(\hat{t}_{k,j}) \int_{\min\{\hat{t}_{k,j}, t_{k,j+1}\}}^{t_{k,j+1}} \lambda_{k,s}(u)\mathrm{d}u, \end{aligned}$$

Here, we introduce two indicator variables: (1) $Y_{k,s,j} = 1$ if transition type s is observed between the j^{th} and $(j+1)^{\text{th}}$ interviews, and (2) $R_{k,s}(t) = 1$ if the individual is exposed to the risk of transition type s at time t.

Estimation under Left Truncation & Censoring I



- When an individual joined the survey after the age of 50 and he/she was not in a functionally disabled state, we cannot observe
  1. $\mathbb{1}_F(t_1)$: presence of past functional disability
  2. T_{t_1}: the latest transition time before the first interview (if any)
- We use an EM algorithm to find maximum likelihood estimates in the presence of missing values.

Estimation under Left Truncation & Censoring II

MCEM-algorithm for Hawkes process

1. Initialize $\theta^{(1)}$: We initialize the parameters assuming no truncation.
2. For $i = 1, 2, 3, \ldots,$ iterate E-step and M-step until convergence
   - **2.1 E-step:** Since analytical solution is unavailable, we perform Monte Carlo approximation to obtain the Q value:

     $$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(i)}) = \mathbb{E}_{\mathbb{1}_F, \mathcal{T}_{trunc}|data,\boldsymbol{\theta}^{(i)}}\left[l(\boldsymbol{\theta})\right] = \mathbb{E}_{\mathbb{1}_F|data,\boldsymbol{\theta}^{(i)}}\left[\mathbb{E}_{\mathcal{T}_{trunc}|\mathbb{1}_F,data,\boldsymbol{\theta}^{(i)}}\left[l(\boldsymbol{\theta})\right]\right] \quad (3)$$

     We use 10,000 simulated individual's health transition history sampled from $\boldsymbol{\theta}^{(i)}$.
   - **2.2 M-step:** We use numerical optimization algorithm to obtain the next estimates[2]

[2] We use the quasi-Newton method for numerical optimization.

Results

Estimation Results I. Goodness of Fits: LRT

Table 1. Likelihood-ratio test results of health transition models. Prototype model-E and Prototyp model-R refer to self-exciting health transition models employing exponential and Rayleigh kernels respectively. In the prototype models, each specification denotes the type of transition to whicl the self-exciting kernel is applied.

Null	Alternative	Degrees of freedom	Test statistic
	Prototype model-E		
	recovery	2	213.3***
	recurrent disability	2	2,020.3***
Baseline model	disabled mortality	2	48.5***
	reactivated mortality	2	46.8***
	Prototype model-R		
	recovery	3	1,405.0***
	recurrent disability	3	2,784.8***
Baseline model	disabled mortality	3	645.9***
	reactivated mortality	3	121.3***

Estimation Results I. Goodness of Fits: LRT

Prototype model-E			
recovery	Full model-E	6	2,143.4***
recurrent disability	Full model-E	6	336.4***
disabled mortality	Full model-E	6	2,308.2***
reactivated mortality	Full model-E	6	2,309.9***
Prototype model-R			
recovery	Full model-R	9	3,518.3***
recurrent disability	Full model-R	9	2,138.5***
disabled mortality	Full model-R	9	4,277.4***
reactivated mortality	Full model-R	9	4,802.0***

*** p-value < 0.0005.

- Our goodness-of-fit results demonstrate that a health transition history has a significant impact on future health transitions.

Estimation Results I. Goodness of Fits: AIC&BIC

	No. of parameters	AIC	BIC
Baseline model	12	169,437.7	169,533.9
Prototype model-E			
recovery	14	169,228.3	169,340.6
recurrent disability	14	167,421.3	167,533.6
disabled mortality	14	169,393.1	169,505.4
reactivated mortality	14	169,394.8	169,507.1
Full model-E	20	**167,096.9**	**167,257.3**
Prototype model-R			
recovery	15	168,038.7	168,158.9
recurrent disability	15	166,658.8	166,779.1
disabled mortality	15	168,797.8	168,918.0
reactivated mortality	15	169,322.4	169,442.6
Full model-R	24	**164,538.3**	**164,730.8**

Note: The lowest values of the AIC and BIC in each of the bottom two panels are highlighted in bold.

- The Rayleigh kernel, where the past transition effect does not decay immediately following a transition, has a better goodness-of-fit than the exponential kernel.

Estimation Results II. Estimated Kernels

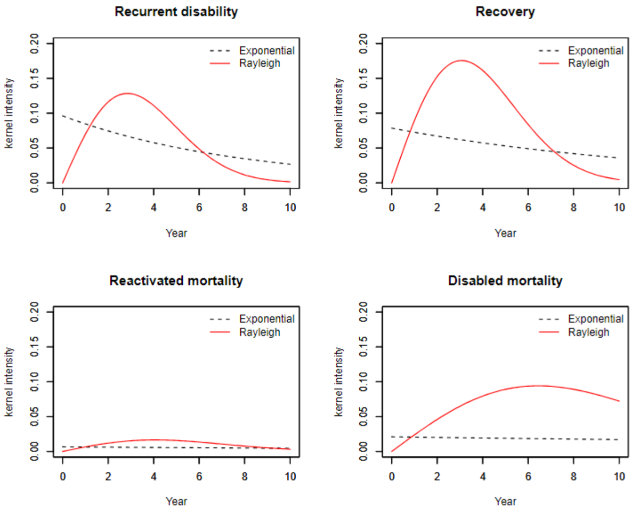


Figure 4. Estimated Hawkes kernels for exponential and Rayleigh kernels

Estimation results III. Future Life Expectancy

Table 4. Model implied future lifetime statistics for women by health status at age 65: mean and standard deviation (SD). The simulation starts with those who are in the state healthy at age 50. The maximum attainable age is 110. Full model-E and Full model-R indicate the full self-exciting models with exponential and Rayleigh kernels, respectively.

Female	Alive at 65			Non-disabled at 65			Disabled at 65		
	Baseline	Full model -E	Full model -R	Baseline	Full model -E	Full model -R	Baseline	Full model -E	Full model -R
Total future lifetime									
Mean	18.81	18.91	19.86	19.07	19.31	20.17	15.85	14.83	16.98
SD	9.30	9.09	9.13	9.12	8.99	9.05	10.23	9.39	9.24
Non-disabled future lifetime									
Mean	16.04	15.71	15.86	16.22	15.99	15.85	14.11	12.74	15.37
SD	8.73	8.70	8.81	8.62	8.72	8.78	9.21	8.38	8.88
Disabled future lifetime									
Mean	2.76	3.20	4.00	2.85	3.31	4.32	1.74	2.09	1.61
SD	4.02	4.96	6.08	4.06	5.03	6.30	3.36	4.02	3.48
Non-disabled lifetime over total future lifetime									
Mean	0.86	0.84	0.82	0.86	0.84	0.81	0.92	0.89	0.92
SD	0.20	0.23	0.25	0.20	0.23	0.26	0.15	0.19	0.17
Age at onset of disability†									
Mean	72.13	74.04	74.01	73.52	76.24	77.06	61.05	60.03	59.49
SD	11.71	11.71	11.92	11.58	11.02	10.65	3.50	3.91	4.01

† Age at onset of disability for individuals who become functionally disabled after turning 50.

Estimation Results IV. Insurance Pricing

Table 6. Actuarially fair lump-sum premiums for insurance products calculated from the simulated health trajectories by subscription age and health status. Full model-E and Full model-R indicate the full self-exciting models with exponential and Rayleigh kernels, respectively

Subscription age (Difference from)	Female Baseline	Female Full model -E	Female Full model -R	Male Baseline	Male Full model -E	Male Full model -R
$1,000/month life annuity sold to a non-disabled individual (unit: $1,000)						
65	174.21	176.24	182.60	153.57	157.03	159.94
(Baseline)		1.16%	4.82%		2.25%	4.15%
(Full model-E)			3.61%			1.85%
75	122.93	123.65	129.28	104.80	105.63	107.64
(Baseline)		0.59%	5.16%		0.78%	2.71%
(Full model-E)			4.55%			1.91%
$1,000/month life annuity sold to a disabled individual (unit: $1,000)						
65	146.49	139.62	157.63	122.53	118.66	134.51
(Baseline)		-4.69%	7.61%		-3.16%	9.78%
(Full model-E)			12.90%			13.36%
75	93.25	93.47	108.50	76.10	73.28	85.35
(Baseline)		0.24%	16.36%		-3.71%	12.15%
(Full model-E)			16.08%			16.47%

Estimation Results IV. Insurance Pricing

\$100/day LTCI sold to a non-disabled individual (unit: \$1,000)						
65	74.41	83.02	105.80	44.27	51.02	67.05
(Baseline)		11.57%	42.18%		15.24%	51.47%
(Full model-E)			27.43%			31.43%
75	69.91	72.11	86.48	40.68	41.30	49.78
(Baseline)		3.15%	23.71%		1.52%	22.37%
(Full model-E)			19.92%			20.53%

- LTCI is notoriously difficult to price, and our simulations suggest that the premium is extremely sensitive to different model assumptions.

Estimation Results IV. Insurance Pricing

Life care annuity sold to a non-disabled individual (unit: $1,000)						
65	248.62	259.26	288.40	197.84	208.05	226.99
(Baseline)		4.28%	16.00%		5.16%	14.74%
(Full model-E)			11.24%			9.11%
75	192.84	195.76	215.75	145.48	146.92	157.42
(Baseline)		1.52%	11.88%		0.99%	8.21%
(Full model-E)			10.21%			7.15%
Life care annuity sold to a disabled individual (unit: $1,000)						
65	253.60	277.38	289.24	212.48	235.18	255.73
(Baseline)		9.38%	14.06%		10.68%	20.36%
(Full model-E)			4.28%			8.74%
75	193.22	211.28	221.21	160.13	166.73	183.74
(Baseline)		9.34%	14.48%		4.12%	14.75%
(Full model-E)			4.70%			10.21%

- Bundling LTCI with life annuities (life care annuity) can potentially reduce the impact of model misspecification on LTCI pricing.

Conclusion

Discussions and Conclusions

- We developed a four-state health transition model that accounts for the effects of past functional disabilities on future states.
- Utilizing a self-exciting process, the model effectively captures how recent health transitions influence future transitions.
- Our contributions extend beyond model development to significant improvements in estimation techniques.
- We also calculated insurance pricing for life annuities and long-term care policies, demonstrating how bundling can mitigate risks associated with model misspecification in pricing.

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Questions & Answers

- Contact information: jung320@purdue.edu
- Personal webpage: https://jiwon-jung.github.io/