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Modeling Multi-state Health Transitions with Hawkes Processes

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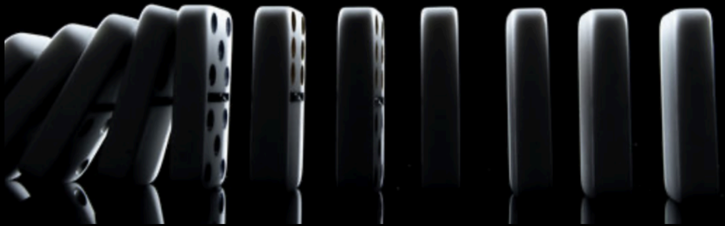
Movements with Momentum



- Most stochastic models used in quantitative finance and insurance assume the Markov property because of its mathematical tractability.
- One commonly observed phenomenon violating the Poisson arrival as well as the Markov assumption is the momentum effect.

Beyond the Markov Models

- Does the concept of the "momentum effect" apply to health transition dynamics?
- To capture this momentum effect, what alternative methods can we use?

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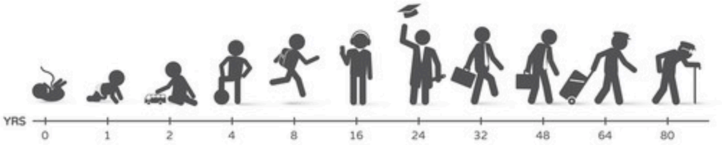
Introduction

Introduction



- Understanding the dynamics of health transition is crucial for pricing aged care products effectively in the evolving health market.
- In particular, impact of functional disability on future transitions has been commonly studied with respect to activities of daily living (ADL) dependencies [1]–[3].

Previous Studies

- [1]–[3] mainly assume the Markov property for modelling health transitions, for which the probabilities of transition at each age depend on the current status only.
- Showing that the probabilities of functional status transitions are duration dependent, another line of research [4], [5] assumes semi-Markov process models to incorporate not only age and the current status but also on the duration in the current state.
- However, the state and duration effect with respect to the past functional disability experience has been less studied.

Motivation

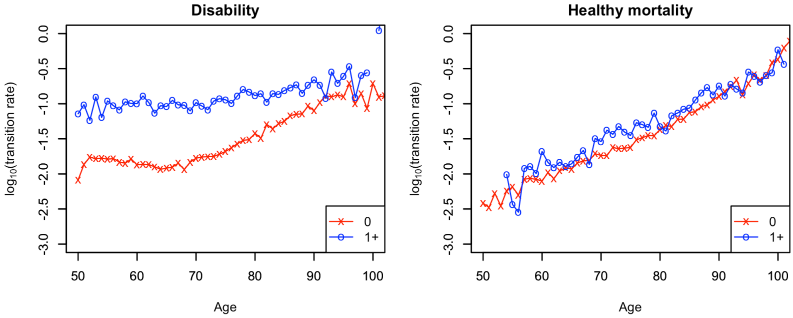
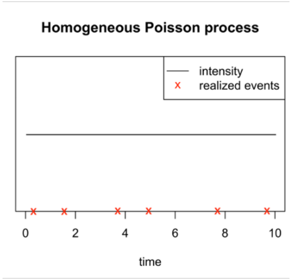


Figure 1. Crude health transition rates with respect to the number of past functional disabilities.

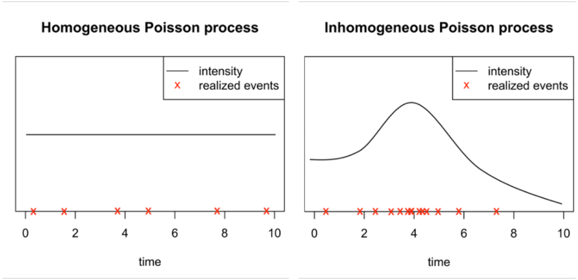
- Our explanatory data analysis suggests that the elderly with prior functional disabilities are at higher risk of experiencing it again and have higher mortality rates than those without a history of disability.

Backgrounds

Momentum and a Hawkes Process



Momentum and a Hawkes Process



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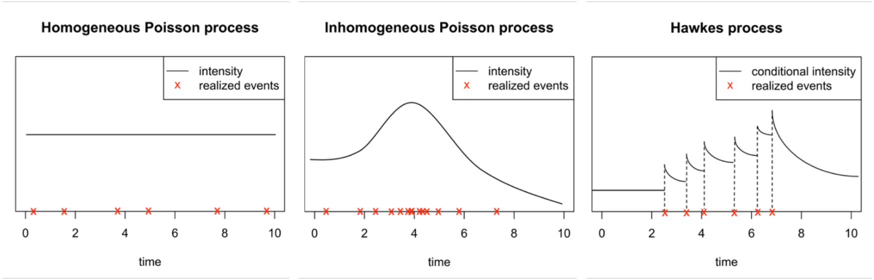
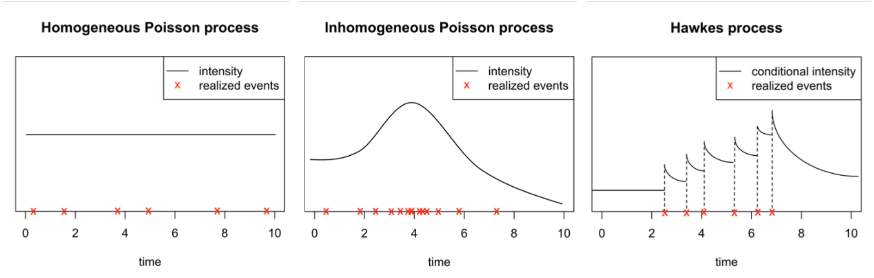


Momentum and a Hawkes Process



- A counting process with a stochastic intensity is called a doubly stochastic Poisson process.
- A Hawkes process [6] is a popular doubly stochastic process with *self-exciting* properties; *an event occurrence increases the probability of the occurrence of another event*.

Momentum and a Hawkes Process

Definition

A Hawkes process is a point process $N(t)$ which is characterized by its conditional intensity $\lambda(t)$ with respect to its natural filtration:

$$\lambda(t|\mathcal{F}_{t-}) = \phi(t) + \int_0^t \mu(t-s)\mathrm{d}N(s), \tag{1}$$

where $\phi(t)$ is the background intensity function, and the $\mu(t)$ is the excitation function satisfying $\int_0^\infty \mu(s)\mathrm{d}s < 1$.

- Hawkes processes model self-exciting properties in diverse fields:
  - Finance: Hawkes [7] and Da Fonseca and Zaatour [8]
  - Insurance: Swishchuk, Zagst, and Zeller [9] and Jung, Lee, and Xu [10]
  - Epidemiology: Browning, Sulem, Mengersen, *et al.* [11]

Goal

- Our goal is to estimate the intensity of age and gender-specific transitions by incorporating the impact of the past functional disability as well as time spent in the current state using a Hawkes process.

Three-State Health Transition Model

Data Preparation I

- We use the `RAND HRS Data` 1992-2018 from the U.S. Health and Retirement Study (HRS), a nationally representative longitudinal panel survey.[1]
- The HRS is a biennial survey which began in 1992 and follows up with interviews of initially non-institutionalised Americans aged 50 and above.
- The health state is determined by a person's ability to perform activities of daily living (ADLs), such as bathing, toileting, and dressing.

[1] https://hrs.isr.umich.edu/data-products

Data Preparation II

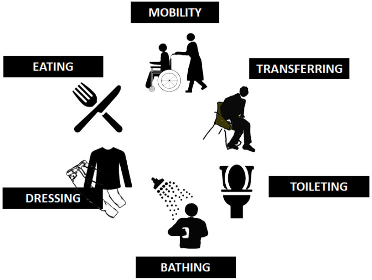


Figure 2. Six activities of daily livings (ADLs) (credit: [12])

- Two or more ADL dependencies indicate functional disability, in line with long-term care insurers' practice.

Three-State Health Transition Model I

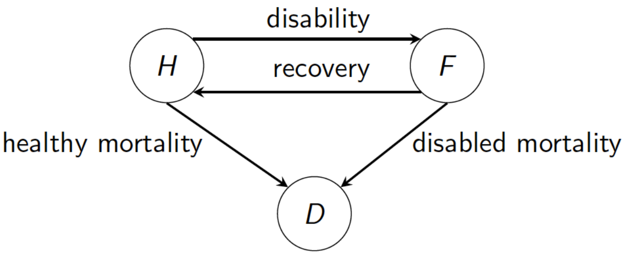


Figure 3. The three-state health transition model: H, F, and D denote healthy, functionally disabled, and dead states, respectively.

Three-State Health Transition Model II

- The transition intensity for individual k of transition type $s \in \{1, 2, 3, 4\}$ at time t is given by

$$\lambda_s(t) = \underbrace{\phi_s(t)}_{\text{background intensity}} + \underbrace{\mu_s(t - T_t)}_{\text{exciting function}} \cdot \underbrace{\mathbb{1}_F(t)}_{\text{disability indicator}},$$

  - $\phi_s(t)$ captures the impact of observable variates such as the (scaled) age $x_k(t)$ and the gender indicator F_k at time t.
  - $\mu_s(\cdot)$ captures the impact of the past functional disability and the duration in the current state ($t - T_t$, where T_t is the latest transition time).
  - $\mathbb{1}_F(t) = 1$ if functionally disabled at least once before time t.

Three-State Health Transition Model III

- Choice of Hawkes kernels $\mu_s(\cdot)$:
  - Exponential kernel (monotonic decay):

$$\mu_s(x) = \alpha_s \, e^{-\delta_s x}, \quad \alpha_s \geq 0, \delta_s > 0, \alpha_s < \delta_s.$$

  - Rayleigh kernel (non-monotonic decay):

$$\mu_s(x) = \theta_s(x + \kappa_s) \, e^{-\eta_s (x+\kappa_s)^2/2}, \quad \theta_s \geq 0, \eta_s > 0, \kappa_s > 0, \theta_s < \eta_s.$$

Estimation

Maximum Likelihood Estimation

Suppose there are a total of K individuals, S transition types, and J interview waves. The complete log likelihood function is given by

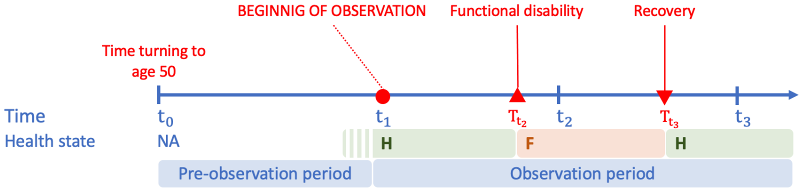
$$l(\boldsymbol{\theta}) = \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{j=1}^{J-1} l_{k,s,j}(\boldsymbol{\theta}), \tag{2}$$

where $\boldsymbol{\theta}$ denotes the set of parameters to be estimated, and

$$\begin{aligned} l_{k,s,j}(\boldsymbol{\theta}) = Y_{k,s,j} \ln \lambda_{k,s}(\hat{t}_{k,j}) - R_{k,s}(t_{k,j}) \int_{t_{k,j}}^{\min\{\hat{t}_{k,j}, t_{k,j+1}\}} \lambda_{k,s}(u)\mathrm{d}u \\ - R_{k,s}(\hat{t}_{k,j}) \int_{\min\{\hat{t}_{k,j}, t_{k,j+1}\}}^{t_{k,j+1}} \lambda_{k,s}(u)\mathrm{d}u, \end{aligned}$$

Here, we introduce two indicator variables: (1) $Y_{k,s,j} = 1$ if transition type s is observed between the j^{th} and $(j+1)^{\text{th}}$ interviews, and (2) $R_{k,s}(t) = 1$ if the individual is exposed to the risk of transition type s at time t.

Estimation under Left Truncation & Censoring I



- When an individual joined the survey after the age of 50 and he/she was not in a functionally disabled state, we cannot observe
  1. $\mathbb{1}_F(t_1)$: presence of past functional disability
  2. T_{t_1}: the latest transition time before the first interview (if any)
- We use an EM algorithm to find maximum likelihood estimates in the presence of missing values.

Estimation under Left Truncation & Censoring II

EM-algorithm for Hawkes process

1. Initialize $\theta^{(1)}$: We initialize the parameters assuming no truncation.
2. For $i = 1, 2, 3, \ldots,$ iterate E-step and M-step until convergence
   2.1 **E-step:** Since analytical solution is unavailable, we perform Monte Carlo approximation to obtain the Q value:

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(i)}) = \mathbb{E}_{\mathbb{1}_F, \mathcal{T}_{trunc}|data, \boldsymbol{\theta}^{(i)}}\left[l(\boldsymbol{\theta})\right] = \mathbb{E}_{\mathbb{1}_F|data, \boldsymbol{\theta}^{(i)}}\left[\mathbb{E}_{\mathcal{T}_{trunc}|\mathbb{1}_F, data, \boldsymbol{\theta}^{(i)}}\left[l(\boldsymbol{\theta})\right]\right] \tag{3}$$

   We use 10,000 simulated individual's health transition history sampled from $\boldsymbol{\theta}^{(i)}$.

   2.2 **M-step:** We use numerical optimization algorithm to obtain the next estimates[2].

[2]We use `optim` function in `R`

Results

Estimation Results I. Goodness of Fits: LRT

Table 1. Likelihood ratio test results of health transition models. Hawkes-E and Hawkes-R indica
the Hawkes health transition models with exponential and Rayleigh kernels, respectively.

Null	Alternative	Degrees of freedom	Test statistic
	Single Hawkes-E		
	disability	2	2,020.3***
Baseline model	recovery	2	213.3***
	healthy mortality	2	46.8***
	disabled mortality	2	48.5***
	Single Hawkes-R		
	disability	3	2,784.8***
Baseline model	recovery	3	1,405.0***
	healthy mortality	3	121.3***
	disabled mortality	3	645.9***
Single Hawkes-E			
disability		6	336.4***
recovery	Full Hawkes-E	6	2,143.4***
healthy mortality		6	2,309.9***
disabled mortality		6	2,308.2***
Single Hawkes-R			
disability		9	2,138.5***
recovery	Full Hawkes-R	9	3,518.3***
healthy mortality		9	4,802.0***
disabled mortality		9	4,277.4***

*** p-value < 0.0005.

Estimation Results I. Goodness of Fits: AIC&BIC

Table 2. AIC and BIC statistics of health transition models. Hawkes-E and Hawkes-R indicate the Hawkes health transition models with exponential and Rayleigh kernels, respectively.

	No. of parameters	AIC	BIC
Baseline model	12	169,437.7	169,533.9
Single Hawkes-E			
disability	14	167,421.3	167,533.6
recovery	14	169,228.3	169,340.6
healthy mortality	14	169,394.8	169,507.1
disabled mortality	14	169,393.1	169,505.4
Full Hawkes-E	20	**167,096.9**	**167,257.3**
Single Hawkes-R			
disability	15	166,658.8	166,779.1
recovery	15	168,038.7	168,158.9
healthy mortality	15	169,322.4	169,442.6
disabled mortality	15	168,797.8	168,918.0
Full Hawkes-R	24	**164,538.3**	**164,730.8**

Estimation Results II. Estimated Kernels

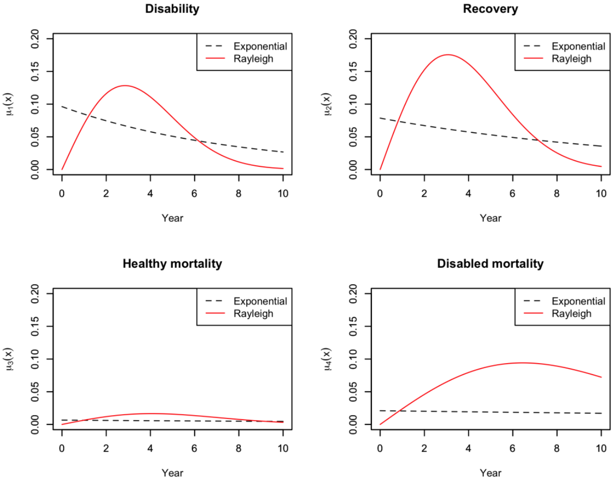


Figure 4. Estimated Hawkes kernels for exponential and Rayleigh kernels

Estimation results III. Future Life Expectancy

Table 4. Model implied future life expectancy for females conditioned on their health status at age 65 including mean and standard deviation (SD). The simulation starts with those who are in a healthy state at age 50. The maximum attainable age is 110. Hawkes-E and Hawkes-R indicate the full Hawkes models with exponential and Rayleigh kernels, respectively.

	All at 65			Healthy at 65			Disabled at 65		
	Baseline	Hawkes -E	Hawkes -R	Baseline	Hawkes -E	Hawkes -R	Baseline	Hawkes -E	Hawkes -R
Total future life expectancy									
Mean	18.81	18.91	19.86	19.07	19.31	20.17	15.85	14.83	16.98
SD	9.30	9.09	9.13	9.12	8.99	9.05	10.23	9.39	9.24
Healthy future life expectancy									
Mean	16.04	15.71	15.86	16.22	15.99	15.85	14.11	12.74	15.37
SD	8.73	8.70	8.81	8.62	8.72	8.78	9.21	8.38	8.88
Disabled future life expectancy									
Mean	2.76	3.20	4.00	2.85	3.31	4.32	1.74	2.09	1.61
SD	4.02	4.96	6.08	4.06	5.03	6.30	3.36	4.02	3.48
Healthy over total future life expectancy									
Mean	0.86	0.84	0.82	0.86	0.84	0.81	0.92	0.89	0.92
SD	0.20	0.23	0.25	0.20	0.23	0.26	0.15	0.19	0.17
Age at onset of disability†									
Mean	72.13	74.04	74.01	73.52	76.24	77.06	61.05	60.03	59.49
SD	11.71	11.71	11.92	11.58	11.02	10.65	3.50	3.91	4.01

† Age at onset of disability after the age of 50.

Estimation Results IV. Insurance Pricing

Table 6. Model implied lump-sum premiums for insurance products calculated from the simulated health trajectories conditioned on their subscription age and health status. Hawkes-E and Hawkes-R indicate the full Hawkes models with exponential and Rayleigh kernels, respectively

	Female			Male		
Subscription age (Difference from)	Baseline	Hawkes-E	Hawkes-R	Baseline	Hawkes-E	Hawkes-R
$1,000/month life annuity sold to a healthy individual (unit: $1,000)						
65	174.21	176.24	182.60	153.57	157.03	159.94
(Baseline)		1.16%	4.82%		2.25%	4.15%
(Hawkes-E)			3.61%			1.85%
75	122.93	123.65	129.28	104.80	105.63	107.64
(Baseline)		0.59%	5.16%		0.78%	2.71%
(Hawkes-E)			4.55%			1.91%
$1,000/month life annuity sold to a disabled individual (unit: $1,000)						
65	146.49	139.62	157.63	122.53	118.66	134.51
(Baseline)		-4.69%	7.61%		-3.16%	9.78%
(Hawkes-E)			12.90%			13.36%
75	93.25	93.47	108.50	76.10	73.28	85.35
(Baseline)		0.24%	16.36%		-3.71%	12.15%
(Hawkes-E)			16.08%			16.47%

Estimation Results IV. Insurance Pricing

$100/day long term care for disability sold to a healthy individual (unit: $1,000)						
65	74.41	83.02	105.80	44.27	51.02	67.05
(Baseline)		11.57%	42.18%		15.24%	51.47%
(Hawkes-E)			27.43%			31.43%
75	69.91	72.11	86.48	40.68	41.30	49.78
(Baseline)		3.15%	23.71%		1.52%	22.37%
(Hawkes-E)			19.92%			20.53%
Life annuity+LTCI for disability sold to a healthy individual (unit: $1,000)						
65	248.62	259.26	288.40	197.84	208.05	226.99
(Baseline)		4.28%	16.00%		5.16%	14.74%
(Hawkes-E)			11.24%			9.11%
75	192.84	195.76	215.75	145.48	146.92	157.42
(Baseline)		1.52%	11.88%		0.99%	8.21%
(Hawkes-E)			10.21%			7.15%
Life annuity+LTCI for disability sold to a disabled individual (unit: $1,000)						
65	253.60	277.38	289.24	212.48	235.18	255.73
(Baseline)		9.38%	14.06%		10.68%	20.36%
(Hawkes-E)			4.28%			8.74%
75	193.22	211.28	221.21	160.13	166.73	183.74
(Baseline)		9.34%	14.48%		4.12%	14.75%
(Hawkes-E)			4.70%			10.21%

Conclusion

Discussions and Conclusions

- We proposed and estimated a three-state health transition model that incorporates the impact of a previous functional disability.
- Since future health transitions are influenced by recent transitions, a Hawkes process is a natural choice to model health transitions.
- Our health transition model using a Hawkes process effectively addressed the effect of health transition history on future health transitions.
- We calculated insurance prices for a life annuity and a long-term care policy using simulated health transitions.

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Questions & Answers

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