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Attention-Based Reading, Highlighting, and Forecasting of the Limit Order Book

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Market Microstructure

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- **macro vs. micro** $\leftarrow$ earth from the space vs. surface of the land

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- **macro vs. micro** $\leftarrow$ earth from the space vs. surface of the land
  - Classical asset pricing models used in financial engineering have been primarily focused on macro-level market movement models.
  - The macro-movement refers to daily, weekly, and monthly closing price behavior, and the micro-movement refers to the transactional (trade-by-trade) price behavior.
  - **High-frequency trading (HFT), algorithmic trading**

Market Order vs. Limit Order

Market Order (MO)

An order to buy/sell an asset at the best available price

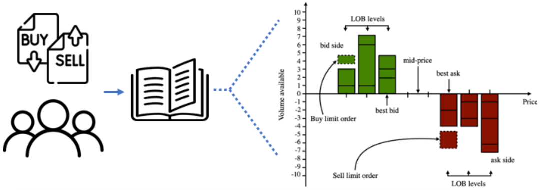
- pros: immediate execution, (i.e., no execution risk),
- cons: pay spread and an additional fee (f), liquidity risk.

Limit Order (LO)

An order to buy/sell an asset at a specified price and quantity

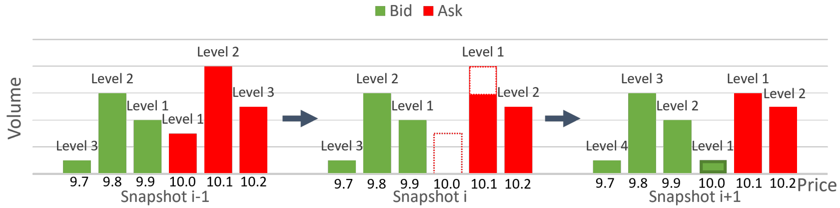
- pros: get better price (when executed) and an additional rebate (r),
- cons: execution risk (the order may never be executed).

Limit Order Book (LOB)



- LOB: Price & time prioritized collection of buy and sell quotes.
- FIFO (first-in-first-out) policy for orders on the same price level.

Limit Order Book (LOB)



	price	size
Ask	10.2	500
	10.1	800
	10.0	300
Bid	9.9	400
	9.8	600
	9.7	100

	price	size
Ask	10.3	200
	10.2	500
	10.1	600
Bid	9.9	400
	9.8	600
	9.7	100

	price	size
Ask	10.3	200
	10.2	500
	10.1	600
Bid	10.0	100
	9.9	400
	9.8	600

Figure 1. A market buy order with size 500 and a limit bid order with size 100 at 10.0 in sequence.

Unique Features of Limit Order Book Dynamics

- Multi-dimension ($2 * 2 * K$ **variables** per snapshot)
  - Ask & Bid
  - Price & Size[1]
  - K-level (ex. $K = 1, 3, 5$, or 10)
- Order priority based on time of arrival
  - **Timestamps** of resolution at least milliseconds and up to nanoseconds depending on the period requested
  - Order flow, tracking the sequence of incoming buy and sell orders

[1] Depth of market, displaying liquidity at different price levels

Previous Studies I

Mid-price forecasting:

- Ntakaris et al. (2018): regression-based approach for high-frequency limit order markets' mid-price prediction.
- Kercheval and Y. Zhang (2015): support vector machines for modeling whether a stock's mid-price increases, decreases, or remains the same;
- Sirignano (2019): a spatial neural network approach to capture the joint distribution of the state of the limit order book at a future time conditional on the current state of the limit order book.
- Z. Zhang et al. (2019): *DeepLOB* architecture utilizes convolutional filters to capture the spatial structure of the LOBs as well as long short-term memory modules to capture longer time dependencies.

Previous Studies II

Optimal execution:

- Park and Van Roy (2015) and Nevmyvaka et al. (2006): machine learning approaches (reinforcement learning) for optimal order execution

Attention-Based Multi-level Limit Order Book Analysis

Motivation

- Limit order book has a unique and complicated structure: price, volume, level, time features
- Due to this complexity, many studies have simplified LOB prediction problems into mid-price forecasting/classifications.
- However, mid-prices have partial information on the LOB dynamics
  - BAS, depths, and levels that affect the dynamics are ignored.

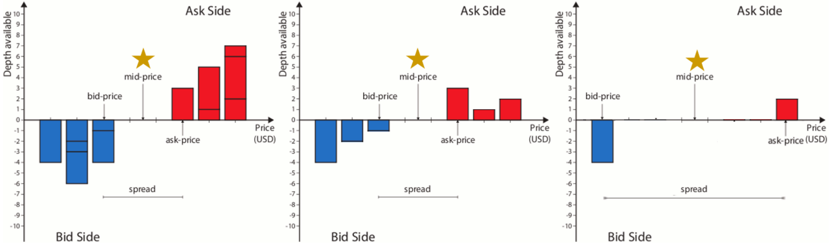


Figure 2. Three different LOB snapshots with the same mid-price (Figure modified from Fernandez-Tapia (2015))

Our Goal

- Our aim is to explore and predict the dynamic nature of limit order book data in its entirety, without simplifying the analysis solely to mid-price movements.

Problem Formulation I

We use $\vec{B}_i$ to denote the vector of the i^{th} bid orders and $\vec{A}_i$ to denote that of the i^{th} ask orders for each snapshot $i \in \{1, \ldots, N\}$ for each level $k \in \{1, \ldots, K\}$. Specifically,

$$\vec{B}_i = \left[t_i, (p^b_{1i}, v^b_{1i}), \ldots, (p^b_{Ki}, v^b_{Ki}) \right], \tag{2.1}$$

where t_i is the time at which snapshot i was captured, p^b_{ki} is the level-k bid price, and $v^b_{ki}(t)$ is the level-k bid volume. Similarly, an ask order is represented as

$$\vec{A}_i = [t_i, (p^a_{1i}, v^a_{1i}), \ldots, (p^a_{Ki}, v^a_{Ki})], \tag{2.2}$$

where t_i is the time at which snapshot i was captured, p^a_{ki} is the level-k ask price, and $v^a_{ki}(t)$ is the level-k ask volume. i represents the number

Problem Formulation II

of a snapshot, and k represents the level of an order. N is the total number of orders. We fix $K = 5$ to obtain the same number of levels both in the bid and ask orders. Based on the context points $i = T - c, \ldots, T$, we predict the future bid and ask order movements of the target points $i = T + 1, \ldots, T + h$.

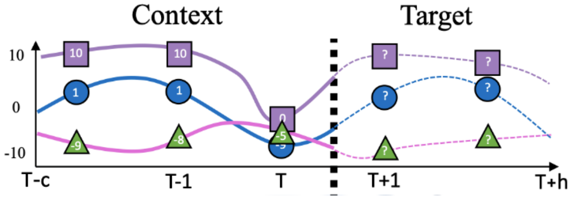


Figure 3. credit: Grigsby et al. (2021)

Our Method

Multi-level time-series prediction using *spacetimeformer* (Grigsby et al., 2021)

- Transformer/Attention-based (Vaswani et al., 2017)
- Multi-level sequence-to-sequence

The attention mechanism uses d-dimensional vectors $Z \in \mathbb{R}^{L_z \times d}$ to update token representation in $X \in \mathbb{R}^{L_x \times d}$ to determine the attention matrix:

$$\text{Attention}(Q, K, V) \in \mathbb{R}^{L_x \times d} = \text{softmax}\left(\frac{QK^T}{\sqrt{d}}\right) V, \tag{2.3}$$

where the query vectors $Q = W^Q X$ and the key vectors $K = W^K Z$ are determined by the learned parameters W^Q and W^K, while the value vectors $V = W^V Z$ are generated with the weights W^V.

Datasets I

- LOBSTER[2]
  - A single day data from 5 different stocks are publicly available
  - Timestamps are available.
- FI-2010[3]
  - 10 days of limit orders with milliseconds time resolution from 5 Nasdaq Nordic stocks
  - Timestamps are not available in the public dataset (event-based)
  - Only normalized values are available
  - Class labels (up/stable/down) are annotated.

[2] https://lobsterdata.com/

[3] https://etsin.fairdata.fi/dataset/73eb48d7-4dbc-4a10-a52a-da745b47a649

Datasets II

			Level 1				Level 2				...
			Ask		Bid		Ask		Bid		
Timestamp	Mid-price	Spread	Price	Quantity	Price	Quantity	Price	Quantity	Price	Quantity	
1275386347944	126200	200	126300	300	126100	17	126400	4765	126000	2800	...
1275386347981	126200	200	126300	300	126100	17	126400	4765	126000	2800	...
1275386347981	126200	200	126300	300	126100	17	126400	4765	126000	2800	...
1275386348070	126050	100	126100	291	126000	2800	126200	300	125900	1120	...
1275386348070	126050	100	126100	291	126000	2800	126200	300	125900	1120	...
1275386348101	126050	100	126100	291	126000	2800	126200	300	125900	1120	...

Figure 4. Order book example of FI-2010 dataset (Ntakaris et al., 2018)

Spacetimeformer for LOB Pipeline

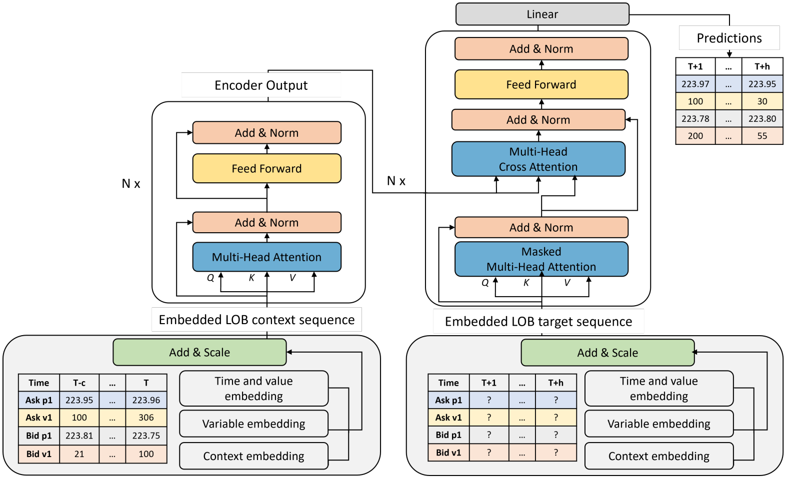


Figure 5. Spacetimeformer (Grigsby et al., 2021) applied to a limit order book

Spacetimeformer for LOB I. Reader

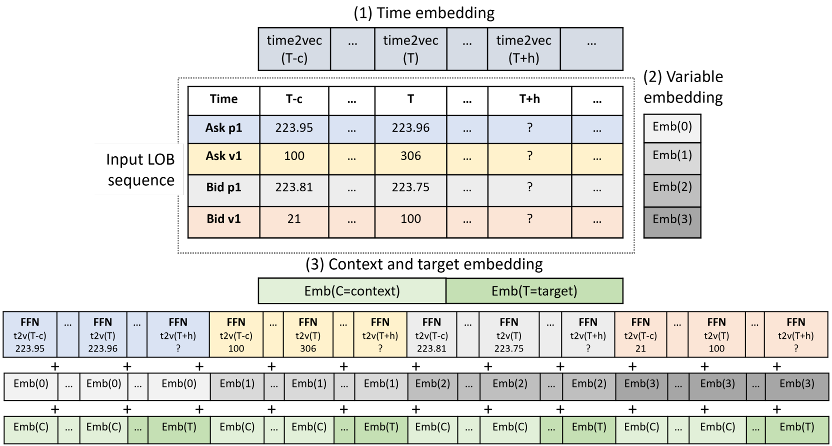
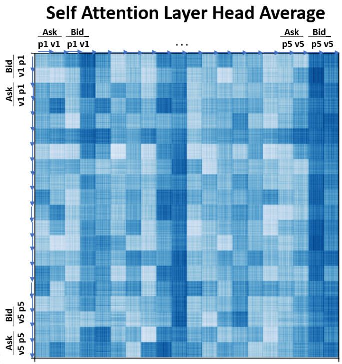
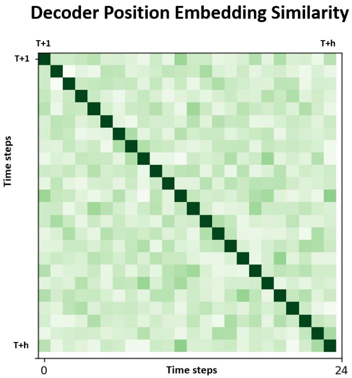


Figure 6. Spatiotemporal embedding incorporates interactions between space/variable, time, and value.

Preliminary results

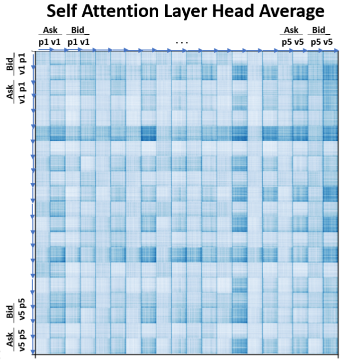
Spacetimeformer for LOB II. Highlighter

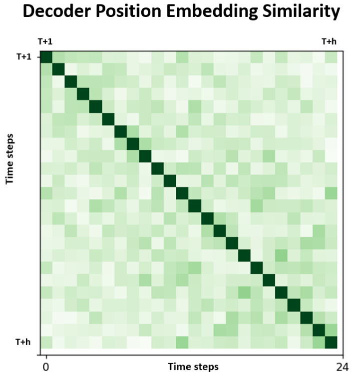




- ……….Epoch #1…………………………………………………………
- Darker shades indicate higher attention scores.

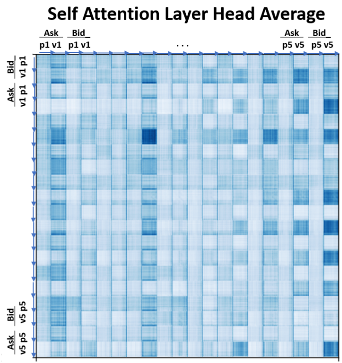
Spacetimeformer for LOB II. Highlighter

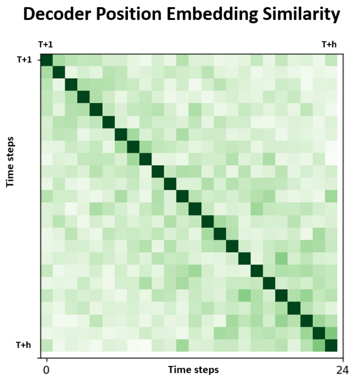




- ....Epoch #3..
- Darker shades indicate higher attention scores.

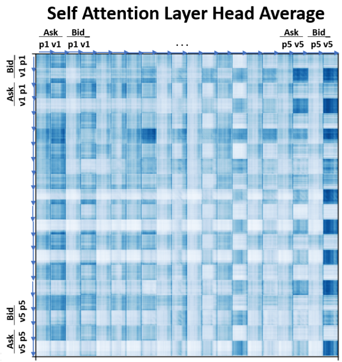
Spacetimeformer for LOB II. Highlighter

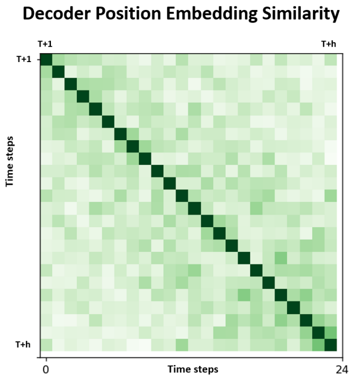




- ……………………………………Epoch #5……………………………
- Darker shades indicate higher attention scores.

Spacetimeformer for LOB II. Highlighter





- ....Epoch #7...............
- Darker shades indicate higher attention scores.

Spacetimeformer for LOB III. Forecaster

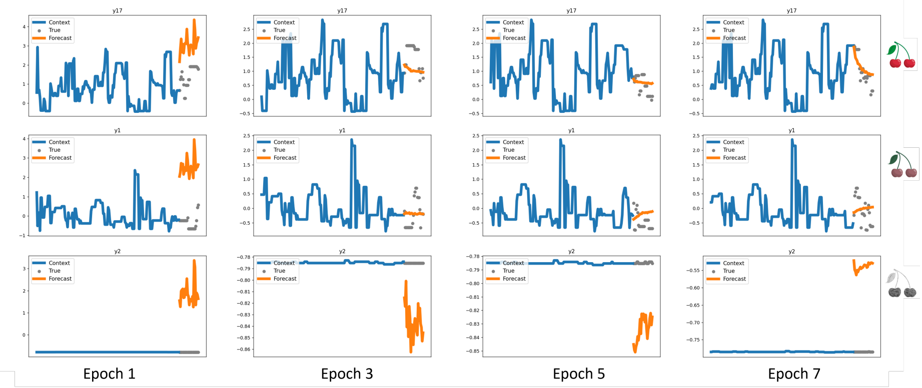


Figure 7. Selected predictions of level-5 ask size (top), level-1 ask size (middle), and level-1 bid price (bottom); Values are normalized and scaled

- The first three days' of FI-2010 were trained, validated, and tested.
- (test) mse= 0.0002; mae = 0.0090; mape = 0.8724

Discussions and Future Directions

Discussions and Future Directions

- We conducted training on a subset of publicly available limit order book (LOB) data to assess the model's performance, and the results were inspiring.
- However, one major challenge we encountered is the ordinal nature of the price variables in the LOB. This presents a significant obstacle as the original structures may not be preserved during the prediction stage.
- Additionally, training the model with a large amount of high-frequency data demands substantial computational resources. To extend the application to the entire dataset, we need to enhance computational efficiency. This will enable us to process and analyze the complete LOB dataset more effectively.

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Questions & Answers

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