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Modeling Multi-state Health Transitions with Hawkes Processes

Jiwon Jung

Ph.D. Student

Department of Statistics

Purdue University

PURDUE UNIVERSITY®

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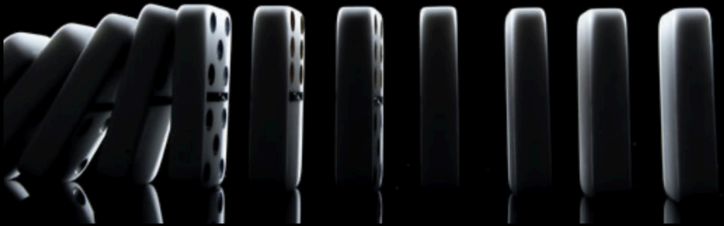
Collaborators

Prof. Kiseop Lee
Department of Statistics
Purdue University

Prof. Mengyi Xu
Department of Statistics
Department of Mathematics
Purdue University

Markets with Momentum



- Most stochastic models used in quantitative finance and insurance assume a Markov property because of its mathematical tractability.
- One commonly observed phenomenon violating the Poisson arrival as well as the Markov assumption is the momentum effect.

Beyond the Markov Models

- Does the concept of "momentum effect" apply to health transition dynamics?
- What alternative methods can be used to capture this momentum effect, beyond the traditional Markov models?

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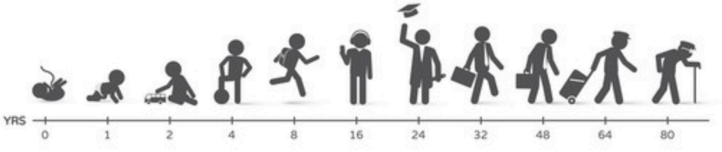


- Understanding the dynamics of health transition is crucial for pricing aged care products effectively in the evolving health market.
- In particular, impact of functional disability on future transitions has been commonly studied with respect to activities of daily living (ADL) dependencies [1]–[3].

Existing literature

- Prior literature [1]–[3] mainly assumes Markov property for modelling health transitions, for which the probabilities of transition at each age depend on the current status only.
- Showing that the probabilities of functional status transitions are duration dependent, another line of research [4], [5] assumes semi-Markov process model to incorporate not only age and the current status but also on the duration in the current state.
- However, the state and duration effect with respect to the past functional disability experience has been less studied.

Motivation

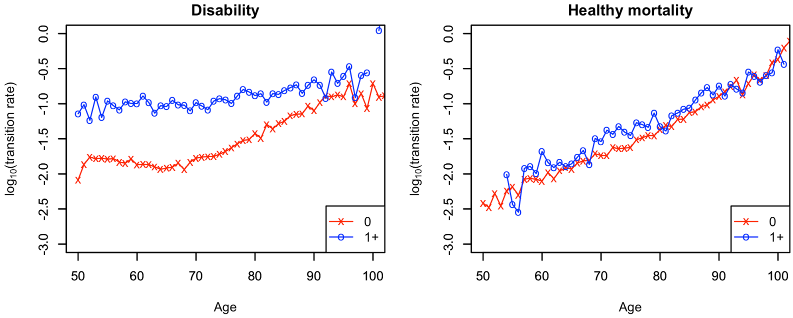
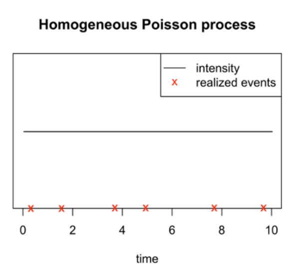


Figure 1. Crude health transition rates with respect to the number of past functional disabilities.

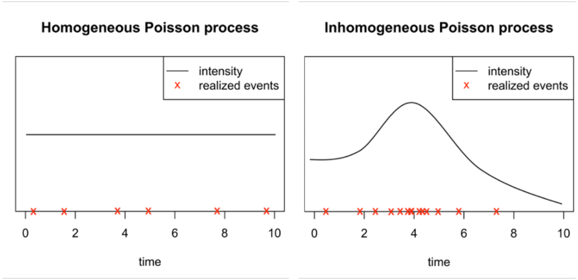
- Our explanatory data analysis suggests that the elderly with prior functional disabilities are at higher risk of experiencing it again and have higher mortality rates than those without a history of disability.

Backgrounds

Markets with Momentum and Hawkes Processes



Markets with Momentum and Hawkes Processes



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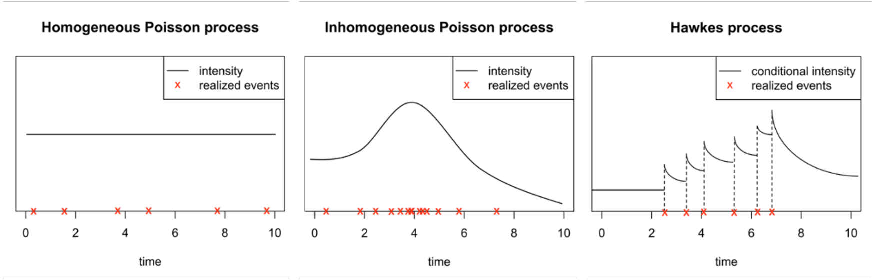
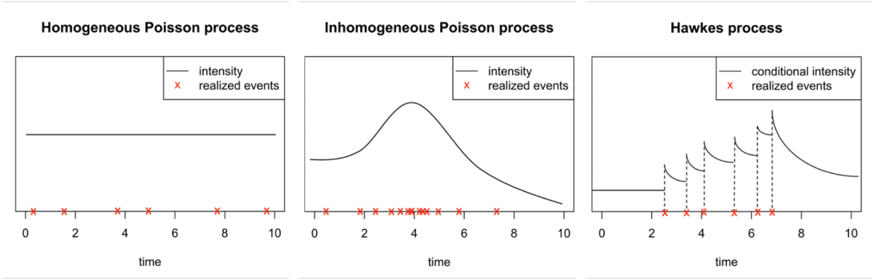


Markets with Momentum and Hawkes Processes



- A counting process with a stochastic intensity is called a doubly stochastic Poisson process.
- A Hawkes process [6] is a popular doubly stochastic process with *self-exciting* properties; *an event occurrence increases the probability of the occurrence of another event*.

Markets with Momentum and Hawkes Processes

Definition

A Hawkes process is a point process $N(t)$ which is characterized by its conditional intensity $\lambda(t)$ with respect to its natural filtration:

$$\lambda(t|\mathcal{F}_{t-}) = \phi(t) + \int_0^t \mu(t-s)N(s), \tag{1}$$

where $\phi(t)$ is the background intensity function, and the $\mu(t)$ is the excitation function satisfying $\int_0^\infty \mu(s)s < 1$.

- Hawkes processes model self-exciting properties in diverse fields:
  - Finance: Hawkes [7] and Da Fonseca and Zaatour [8]
  - Insurance: Swishchuk, Zagst, and Zeller [9] and Jung, Lee, and Xu [10]
  - Epidemiology: Browning, Sulem, Mengersen, *et al.* [11]

Goal

- Our goal is to estimate the intensity of age and gender-specific transitions by incorporating the impact of the past functional disability as well as time spent in the current state using Hawkes processes.

Three-State Health Transition Model

Data Preparation I

- We use the `RAND HRS Data` 1992-2018 from the U.S. Health and Retirement Study (HRS), a nationally representative longitudinal panel survey.[1]
- The HRS is a biennial survey which began in 1992 and follows up with interviews of initially non-institutionalised Americans aged 50 and above.
- The health state is determined by a person's ability to perform activities of daily living (ADLs), such as bathing, toileting, and dressing.

[1] https://hrs.isr.umich.edu/data-products

Data Preparation II

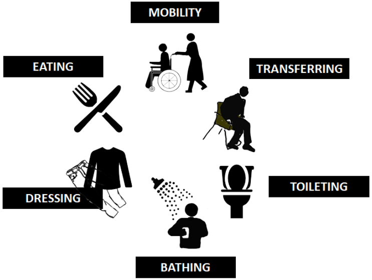


Figure 2. Six activities of daily livings (ADLs) (credit: [12])

- Two or more ADL dependencies indicate functional disability, in line with long-term care insurers' practice.

Three-State Health Transition Model I

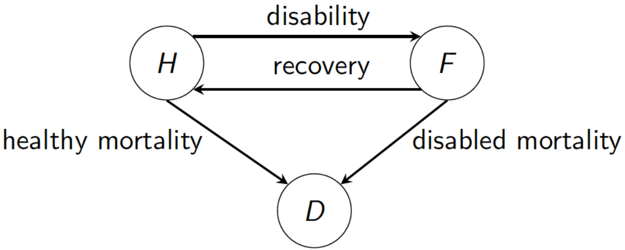


Figure 3. The three-state health transition model: H, F, and D denote healthy, functionally disabled, and dead states, respectively.

Three-State Health Transition Model II

- The transition intensity for individual k of transition type $s \in \{1, 2, 3, 4\}$ at time t is given by

$$\lambda_s(t) = \underbrace{\phi_s(t)}_{\text{background intensity}} + \underbrace{\mu_s(t - T_t)}_{\text{exciting function}} \cdot \underbrace{\mathbb{1}_F(t)}_{\text{disability indicator}},$$

  - $\phi_s(t)$ captures the impact of observable variates such as (scaled) age $x_k(t)$ and gender indicator F_k at time t.
  - $\mu_s(\cdot)$ captures the impact of past functional disabilityand duration in the current state.
  - $\mathbb{1}_F(t) = 1$ if functionally disabled at least once before time t.

Three-State Health Transition Model III

- Choice of Hawkes kernels $\mu_s(\cdot)$:
  - Exponential kernel (monotonic decay):

$$\mu_s(x) = \alpha_s \, e^{-\delta_s x}, \quad \alpha_s \geq 0, \delta_s > 0, \alpha_s < \delta_s.$$

  - Rayleigh kernel (non-monotonic decay):

$$\mu_s(x) = \theta_s(x + \kappa_s) \, e^{-\eta_s(x+\kappa_s)^2/2}, \quad \theta_s \geq 0, \eta_s > 0, \kappa_s > 0, \theta_s < \eta_s.$$

Estimation

Maximum Likelihood Estimation

Suppose there are a total of K individuals, S transition types, and J interview waves. The complete log likelihood function is given by

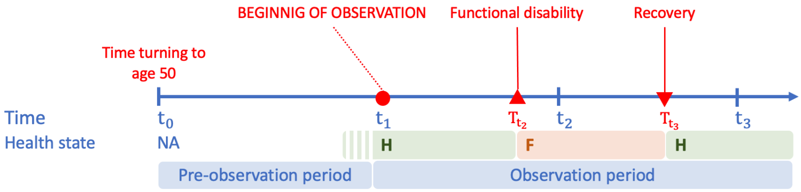
$$l(\boldsymbol{\theta}) = \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{j=1}^{J-1} l_{k,s,j}(\boldsymbol{\theta}), \tag{2}$$

where $\boldsymbol{\theta}$ denotes the set of parameters to be estimated, and

$$\begin{aligned} l_{k,s,j}(\boldsymbol{\theta}) = Y_{k,s,j} \ln \lambda_{k,s}(\hat{t}_{k,j}) - R_{k,s}(t_{k,j}) \int_{t_{k,j}}^{\min\{\hat{t}_{k,j}, t_{k,j+1}\}} \lambda_{k,s}(u)u \\ - R_{k,s}(\hat{t}_{k,j}) \int_{\min\{\hat{t}_{k,j}, t_{k,j+1}\}}^{t_{k,j+1}} \lambda_{k,s}(u)u, \end{aligned}$$

Here, we introduce two indicator variables: (1) $Y_{k,s,j} = 1$ if transition type s is observed between the j^{th} and $(j+1)^{\text{th}}$ interviews, and (2) $R_{k,s}(t) = 1$ if the individual is exposed to the risk of transition type s at time t.

Estimation under Left Truncation & Censoring I



- When an individual joined the survey after the age of 50 and he/she was not in a functionally disabled state, we cannot observe
  1. $\mathbb{1}_F(t_1)$: presence of past functional disability
  2. T_{t_1}: the latest transition time before the first interview (if any)
- We use an EM algorithm to find maximum likelihood estimates in the presence of missing values.

Estimation under Left Truncation & Censoring II

EM-algorithm for Hawkes process

1. Initialize $\theta^{(1)}$: We initialize the parameters assuming no truncation.
2. For $i = 1, 2, 3, \ldots,$ Iterate E-step and M-step until convergence[2]
   2.1 **E-step:** Since analytical solution is unavailable, we perform Monte Carlo approximation to obtain the Q value:

   $$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(i)}) =_{\mathbb{1}_F,\tau_{trunc}|data,\boldsymbol{\theta}^{(i)}} [l(\boldsymbol{\theta})] =_{\mathbb{1}_F|data,\boldsymbol{\theta}^{(i)}} \left[_{\tau_{trunc}|\mathbb{1}_F,data,\boldsymbol{\theta}^{(i)}} [l(\boldsymbol{\theta})]\right] \quad (3)$$

   We use 10,000 simulated individual's health transition history sampled from $\boldsymbol{\theta}^{(i)}$.

   2.2 **M-step:** We use numerical optimization algorithm to obtain the next estimates[3].

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[2] Iterate until the difference between the current and previous Q value is less than 10^{-2}

[3] We use `optim` function in `R`

Results

Estimation Results I. Goodness of Fits

Table 1.

Model	Kernel	$d^{\dagger}$	LRT statistic (df)‡	AIC	BIC
Baseline model	(non-Hawkes)	12	-	169437.7	169533.9
Single Hawkes-E	disability	14	2020.3***(2)	167421.3	167533.6
	recovery	14	213.3***(2)	169228.3	169340.6
	healthy mortality	14	46.8***(2)	169394.8	169507.1
	disabled mortality	14	48.5***(2)	169393.1	169505.4
Full Hawkes-E	all four-transition	20	**336.4*****(6)§	**167096.9**	**167257.3**
Single Hawkes-R	disability	15	2784.8***(3)	166,658.8	166779.1
	recovery	15	1405.0***(3)	168038.7	168158.9
	healthy mortality	15	121.3***(3)	169322.4	169442.6
	disabled mortality	15	645.9***(3)	168797.8	168918.0
Full Hawkes-R	all four-transition	24	**2138.5*****(9)§	**164538.3**	**164730.8**

*** p-value < 0.0005

† Number of parameters

‡ Baseline v. Single Hawkes; Single Hawkes v. Full Hawkes

§ The LRT statistic when tested under the null with the maximal likelihood.

Estimation Results II. Estimated Kernels

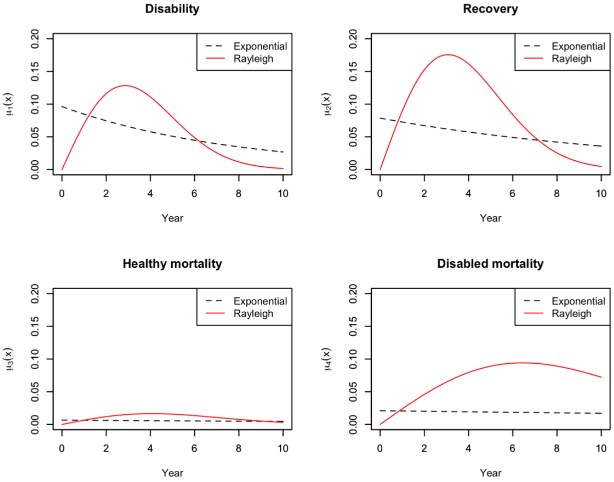

Figure 4. Estimated Hawkes kernels for exponential and Rayleigh kernels

Estimation results III. Future Life Expectancy

Table 2. Model implied future life expectancy for simulated individuals.

	Female			Male		
	Baseline	Hawkes-E	Hawkes-R	Baseline	Hawkes-E	Hawkes-R
Total future life expectancy						
Mean	19.06	19.31	20.23	16.36	16.68	17.17
SD	9.06	8.93	9.02	8.37	8.33	8.34
Healthy future life expectancy						
Mean	16.21	15.95	15.87	14.71	14.73	14.64
SD	8.55	8.67	8.68	8.08	8.21	8.21
Disabled future life expectancy						
Mean	2.85	3.36	4.36	1.65	1.96	2.54
SD	4.09	5.04	6.34	3.04	3.72	4.69
Healthy over total future life expectancy						
Mean	0.86	0.84	0.81	0.90	0.89	0.87
SD	0.20	0.23	0.26	0.18	0.20	0.23
Age at onset of disability[‡]						
Mean	77.89	78.58	78.71	77.03	77.36	77.67
SD	8.39	8.66	8.58	7.70	8.07	8.03

[‡] Age at onset of disability conditional on becoming disabled after the age of 65

Estimation Results IV. Insurance Pricing

Table 3. Model implied lump-sum premiums for insurance products.

	Female			Male		
Subscription age	Baseline	Hawkes-E	Hawkes-R	Baseline	Hawkes-E	Hawkes-R
$1,000/month life annuity sold to a healthy individual						
65	$171,716	$173,600	$180,686	$151,903	$155,174	$157,927
70	$145,539	$146,389	$152,182	$126,021	$128,017	$130,193
75	$118,978	$118,749	$125,476	$101,338	$102,223	$104,280
80	$95,484	$94,383	$98,951	$78,653	$79,156	$81,695
$1,000/month life annuity sold to a disabled individual						
65	$170,471	$153,016	$157,634	$149,361	$133,815	$137,027
70	$145,239	$132,001	$137,576	$125,448	$113,146	$115,839
75	$118,744	$110,367	$114,795	$100,447	$92,459	$94,709
80	$94,696	$88,499	$92,893	$78,427	$73,790	$75,382
$100/day long term care for disability						
55	$69,463	$81,144	$108,198	$42,935	$51,388	$70,386
60	$71,785	$81,890	$107,743	$44,071	$52,135	$70,150
65	$74,411	$83,019	$105,795	$44,268	$51,017	$67,053
70	$72,906	$79,388	$97,556	$44,203	$47,385	$59,723
75	$69,905	$72,110	$86,478	$40,678	$41,297	$49,778
80	$65,196	$62,245	$71,992	$34,720	$33,688	$39,547

Conclusion

Discussions and Conclusions

- We have proposed and estimated a three-state health transition model that incorporates the impact of a previous functional disability.
- Since future health transitions are influenced by recent transitions, a Hawkes process becomes a natural choice to model health transitions.
- Our health transition model using a Hawkes process effectively addresses the effect of health transition histories on future health transitions.
- We calculated a pricing scenario for a life annuity and a long-term care policy using simulated health transitions.

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Questions & Answers

- Contact information: jung320@purdue.edu
- Personal webpage: https://jiwon-jung.github.io/